Authors: Deborah Hett \& Mark Troncale
Title of Lesson: Finding Equivalent Ratios
Date of Lesson: N/A
Length of Lesson: ~60 minutes
Name/Level of Course: Algebra/ 7th Grade

## Appropriateness for Middle School Students:

Students will be paired up in groups of 3 . Each group will be assigned a worksheet. First the instructors will go over section A of the worksheet with the entire class and then the students will be actively engaged because each group must complete the other sections of the worksheet. The students will be working together in their groups trying to come up with the solutions to sections B and C of the worksheet. The students will be dealing with scaling up and down ratios.

## Technology Lesson? No

## Source of the Lesson:

Lappan, Glenda, James T Fey, William M Fitzgerald, Susan N Friel, and Elizabeth Difanis Phillips. Ed. Connected Mathematics 2: Comparing and Scaling. Ratio, Proportion, and Percent. Boston, MA: Pearson, 2009. 31-8. Print.

## Concepts:

A ratio is the relationship between two or more quantities. Moreover, equivalent ratios are those ratios that have the same values when simplified. It is very important to develop strategies for comparing ratios, in order to solve real world problems. Multiplying or dividing the two quantities of a ratio by the same constant will result in an equivalent ratio to the initial ratio.

## Objectives:

- Identify equivalent ratios.
- Calculate equivalent ratios using a common factor to scale a given ratio up or down.
- Develop strategies for comparing ratios.


## Common Core Standards:

- CCSS.Math.Content.7.RP.A. 2 Recognize and represent proportional relationships between quantities.
- CCSS.Math.Content.7.RP.A.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- CCSS.Math.Content.7.RP.A.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- CCSS.Math.Content.7.RP.A.2c Represent proportional relationships by equations. For example, if total cost $t$ is proportional to the number $n$ of items purchased at a constant price $p$, the relationship between the total cost and the number of items can be expressed as $t=p n$.
- CCSS.Math.Content.7.RP.A.2d Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$ where $r$ is the unit rate.


## Math Practices:

- Make sense of mathematics problems and persevere in solving them.
- Attend to Precision

Safety: No safety precautions.

## Materials List and Advanced Preparations:

Students:

- "Scaling Ratios" Worksheet


## Teacher:

- Jeopardy PowerPoint (See accompanying PowerPoint)
- Internet Access (youtube.com)
- Post-Assessment

| ENGAGEMENT <br> Time: 5 Minutes |  |  |
| :---: | :---: | :---: |
| What the Teacher Will Do | Probing/Eliciting Questions | Student <br> Responses and Misconceptions |
| Ask students about their last visit to the zoo. <br> Ask if any of them were ever lucky enough to see the zookeeper feed the animals. <br> Show youtube clip about feeding animals at the zoo. If there are time limitations, watch Start-1.20, 1.51-2.6, and 3.30-3.56. http://www.youtube.com/watch?v=TTgGePpQ7o | How do you think the zookeepers know how much and of what they should feed the animals? <br> How do you think the zookeepers adjust the food portions when more animals are added to the zoo? | [They measure the food using scales.] [They take note of the food that the animals eat in their natural habitat.] <br> [Using scales. <br> They increase the animals' food according to how many animals are added.] |
| Today we are going to look into how zookeeper's determine the amount of food they need to give to the animals ourselves |  |  |


| by looking at ratios. |  |  |
| :--- | :--- | :--- |
| Transition: Let's go ahead and get started. |  |  |


| EXPLORATION <br> Time: 20 Minutes |  |  |
| :---: | :---: | :---: |
| What the Teacher Will Do | Probing/Eliciting Questions | Student Responses and Misconceptions |
| Group students according to the number on their name tents. <br> Pass out scaling ratios worksheet. <br> Write the table "Dietary Needs of Baby Chimp" from the worksheet up on the board. <br> Say: To begin, we will all work on part A together. <br> Use Popsicle stick method to call on students to read. <br> Discuss answers to questions one, two, three, and four. | Ask students how and why they came to conclusions they did. <br> Do you think there are other ways to alter these ratios? <br> How can you use the patterns that you see in your table? <br> Does the pattern give you a way to figure out the ratio for $n$ numbers of baby chimps? Say... 60 ? | [Students should be able to explain the process they are using to convert ratios] Misconception: Ratios can be scaled by ways other than multiplication and division. <br> [If the number represents the number of baby chimps, then we would be able to use the multiplication pattern to find how much feed we'd need for any number of chimps] |
| After going over Part A, instruct students to finish the rest of the worksheet in their groups. <br> Teacher will be monitoring groups progress and answering any questions. <br> Say: Now it is time for you to complete parts b and c of the worksheet in your groups. Everyone in your group should work together to complete it. I will be walking around answering any | Ask students how and why they came to conclusions they did. <br> What differences do you notice in how the mix changes as a chimp moves from being a baby to a young adult? What about from a young adult to an older chimp? | [Students should be able to explain how they are calculating the equivalent ratios] <br> [The ratio of fiber to protein is greater for young adult chimps. i.e. Adult chimps need a lot more fiber.] Misconception: If the fiber to protein ratio is $3: 2$ there is more fiber than if the ratio is $2: 1$. (disprove by |



| EXPLANATION <br> Time: 10 Minutes |  |  |
| :---: | :---: | :---: |
| What the Teacher Will Do | Probing/Eliciting Questions | Student <br> Responses and Misconceptions |
| Use Popsicle stick method to call on students to give their group's responses to the worksheet questions. |  | Students may try to use decimal in their answers. |
| Discuss why ratios can not be given in the form of decimals like fractions can. <br> Ask 3 probing and eliciting questions to the right. <br> Lead discussion on table from part <br> A. Emphasize that we can use | When finding ratios, why do we use fractions and not decimals? | [A ratio compares two numbers, when we have a decimal there is only one number, therefore it does not help us to see the comparison. <br> [You can multiply the entry for 10 chimps |

known information from the table to calculate the amount of food needed for 60 chimps by simply multiplying factors of 60 . We do not always need to start with the formula for one chimp.

Ask next probing questions in sequence with discussions.

Lead discussion on the most efficient method for finding equivalent ratios. This method is to use what we know about equivalent fractions to multiply or divide each part of the ratio by the same number to scale up or down.

Lead a discussion about the common factors of equivalent ratios. Since you are comparing two ratios, you need one of the quantities to be the same in each of the ratios. This is like finding common numerators or common denominators like with fractions. Therefore, we chose a factor to multiply by that will give us the new ratio we want.

Discuss what is meant by scaling and the correct and incorrect ways to scale a ratio.

Write examples up on the board that show the correct and incorrect ways to scaling ratios.

Correct: Multiply or divide by a common factor

Incorrect: Add or subtract, Use decimals, etc.
Looking at the table
from part A, Does the
pattern give you a way
to figure out the ratio for
n numbers of baby
chimps? Say... $60 ?$

Ask if there is only one
way to calculate the
amount of food needed
for 60 chimps?

What is an efficient method of finding a ratio equivalent to a given ratio?

How do you choose what to multiply or divide by?

What does it mean to scale down/up a ratio?

How can we scale down/up a ratio?

## Can you scale

 down/reduce a ratio or fraction by subtracting the same amount from each part of the ratio?by 6 , multiply the entry for 1 chimp by 60, or multiply other factors that give 60, like 20 chimps multiplied by 3.]
[No.] Misconception: Yes, multiply the starting ratio by 60 .
[Use what we know about equivalent fractions to multiply or divide each part of the ratio by the same number to scale up or down.]
[Multiply by a factor that makes the known number equivalent to the desired number.]
[Scaling is shrinking or expanding a number by a factor.] Misconception: Ratios can be altered by adding and subtracting the same amount from each part of the ratio.
[Multiply or divide both parts of the ratio by a constant.]
[NO.]
[No.] Misconception: Yes.

|  | Is 3:2 the same as 2:1? |  |
| :--- | :--- | :--- |
| Show that 2:1 is not the same as |  | [Yes.] |
| $3: 2$ by applying the ratio to a set of |  |  |
| 100 objects. i.e. 3:2 $=60: 40$ and | Is 3:2 the same as 6:4? |  |
| $2: 1 \sim 66: 33$ |  |  |
| Show that these are the same by |  |  |
| making both ratios out of 100. i.e. |  |  |
| $3: 2=60: 40$ and 60:40. |  |  |
| Transition: Now it's time to move |  |  |
| on. |  |  |


| ELABORATION <br> Time: 10 Minutes |  |  |
| :--- | :--- | :--- |
| What the Teacher Will Do | Probing/Eliciting <br> Questions | Student <br> Responses and <br> Misconceptions |
| Split class into 2 groups. (i.e. one side of <br> the class versus the other.) |  |  |
| Show Jeopardy PowerPoint with questions <br> about further applications of scaling and <br> the relationship between fraction <br> equivalence and scaling ratios. <br> (Includes engagement questions from Math <br> Connects: Finding Equivalent Ratios.) |  |  |
| Assign one person the spokesman for the <br> group. That person must give group's <br> response. <br> Have one group start by choosing a <br> category and value. <br> Groups will get 1 minute to answer the <br> questions. <br> Correct answer gives them that number of <br> points. |  |  |
| If they get it wrong discuss correct answer |  |  |
| and no one gets points. |  |  |
| Winning team gets a pat on the back. |  |  |
| Transition: Now it's time to see what we |  |  |

have learned today.

| EVALUATION <br> Time: 10 Minutes |  |  |
| :---: | :---: | :---: |
| What the Teacher Will Do | Probing/Eliciting <br> Questions | Student <br> Responses and <br> Misconceptions |
| Say: We will, now be taking a short quiz. <br> So,return to your desk, and put everything <br> away except a pen or pencil. |  |  |
| Allow students 1 minute to get back to their <br> places. |  |  |
| Say: When you're finished, flip your paper <br> face down, and sit quietly until every one's <br> finished. This is an individual assignment, so <br> there will be no talking once the quiz is <br> passed out. Keep your eyes on your own <br> paper. If you have any questions, raise your <br> hand, and we will come and assist you. |  |  |
| Good Luck!!! <br> Pass out quiz to entire class, and monitor <br> student behavior, answering any questions. <br> Pour glasses of lemonade for each student, <br> while students are testing.. |  |  |
| Pick up quiz once everyone is done. |  |  |
| Say: Great job today guys! |  |  |

## Post-Assessment

Name: $\qquad$
Date : $\qquad$

## Ratios

1. 

a. Draw a line between the following ratios that are equivalent. (Ratios can have more than one or no equivalent.)

| $\frac{7}{13}$ | $\frac{24}{30}$ |
| :--- | :--- |
| $\frac{75}{100}$ | $\frac{14}{20}$ |
| $\frac{20}{68}$ | $\frac{10}{58}$ |
| $\frac{4}{5}$ | $\frac{21}{39}$ |
| $\frac{27}{17}$ | $\frac{3}{4}$ |

b. Explain how you found EACH of the equivalent ratios in part one. Make sure you include the factor that relates the two ratios in your explanation.
2. Find the equivalent ratio by scaling down or up the following ratios. Show all work.
a. Find an equivalent ratio by scaling down the following ratios: (Simplify your answer to the smallest fraction)
i.) $\frac{24}{10}$
ii.) $\frac{136}{256}$
iii.) $\frac{1143}{774}$
b. Find an equivalent ratio by scaling up the following ratios:
i.) $\frac{13}{6}$
ii.) $\frac{21}{9}$
iii.) $\frac{131}{43}$
3. The following table expresses the ratio of water to powder for instant lemonade. Complete the following table.

| Servings | 1 | 2 | 3 | 4 | 5 | 24 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Water (Cups) | 6 |  |  | 24 | 30 |  |
| Powder (Tbs) |  | 2 | 3 |  | 5 |  |

Name: $\qquad$
Date : $\qquad$

## Ratios

a. Draw a line between the following ratios that are equivalent. (Ratios can have more than one or no equivalent.)

| $\frac{7}{13}$ | $\rightarrow$ | $\frac{21}{39}$ | $\frac{24}{30}$ |
| :--- | :--- | :--- | :--- |
| $\frac{75}{100}$ | $\rightarrow$ | $\frac{3}{4}$ | $\frac{14}{20} \rightarrow$ None |
| $\frac{20}{68}$ | $\rightarrow$ | None | $\frac{10}{58} \rightarrow$ None |
| $\frac{4}{5}$ | $\rightarrow$ | $\frac{24}{30}$ | $\frac{21}{39}$ |
| $\frac{21}{28} \rightarrow \frac{3}{4}$ |  | $\frac{3}{4}$ |  |

b. Explain how you found ONE of the equivalent ratio pairs in part one. Make sure you include the constant that relates the two ratios in your explanation.
Answers may vary.
2. Find the equivalent ratio by scaling down or up the following ratios. Show all work.
a. Find an equivalent ratio by scaling down the following ratios (Simplify your answer to the smallest fraction)
i.) $\frac{24}{10}=\frac{12}{5}$
ii.) $\frac{136}{256}=\frac{17}{64}$
iii.) ${ }^{\frac{1143}{774}}=\frac{127}{86}$
b. Find an equivalent ratio by scaling up the following ratios
i.)
$\frac{13}{6}$
ii.) $\quad \frac{21}{9}$
iii.) $\frac{131}{43}$

Answers may vary .
3. The following table expresses the ratio of water to powder for instant lemonade.

Complete the following table.

| Servings | 1 | 2 | 3 | 4 | 5 | 24 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Water (Cups) | 6 | 12 | 18 | 24 | 30 | 144 |
| Powder (Tbs) | $\mathbf{1}$ | 2 | 3 | 4 | 5 | 24 |

Quiz Grading Rubric

| Question/Point Value | Question Breakdown |  |
| :--- | :--- | :--- |
| 1. = 6 points | a. = 4 points <br> b. = 2 points | 1 point per correct ratios connected. <br> 1 point for a correct explanation <br> 1 point for including the correct constant. |
| 2. = 7 points | a. $=3$ points <br> b. $=3$ points <br> Work shown $=1$ point | 1 point per correct equivalent ratio found. |
| 3. = 6 points |  | 1 point per box filled in correctly. |
| Total $=19$ points |  |  |

## Problem 4,3 Scaling Ratios

One of Ming＇s tasks at the county zoo＇s primate house is to mix food for the chimpanzees．The combination of high－fiber nuggets and high－protein nuggets changes as the chimps grow from babies to adults．
Ming has formulas for mixing high－fiber and high－protein nuggets for the chimps．
－Baby chimps： 2 cups high－fiber nuggets and 3 cups high－protein nuggets per serving
－Young adult chimps： 6 cups high－fiber nuggets and 4 cups high－protein nuggets per serving
－Older chimps： 4 cups high－fiber nuggets and 2 cups high－protein nuggets per serving
A．1．What amounts of high－fiber and high－protein nuggets will Ming need when she has to feed 2 baby chimps？ 3 baby chimps？
4 baby chimps？
Copy and complete the table below．
Dietary Needs of Baby Chimps

| Number of Baby Chimps | 1 | 2 | 3 | 4 | 5 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cups of High－Fiber Nuggets | 뇰 | 編 | ＊ | 気 | 捺 | 祹 |
| Cups of High－Protein Nuggets | 縎 | 䋿 | \％ | 彦 | 整 | 3 |

2．What patterns do you see in your table？
3．Ming puts 48 cups of high－protein nuggets into the baby chimp mix． How many cups of high－fiber nuggets does she put into the mix？ Explain．
4．Ming has a total of 125 cups of mix for baby chimps．How many cups of high－fiber nuggets are in the mix？Explain．
B．1．What is the ratio of high－fiber to high－protein nuggets for young adult chimps？
2．Scale this ratio up to show the ratio of high－fiber to high－protein nuggets that will feed 21 young adult chimps．
3．To feed 18 young adults，you need 108 cups of high－fiber nuggets and 72 cups of high－protein nuggets．Show how to scale down this ratio to feed 3 young adult chimps．

Notes
$\qquad$
$\qquad$
C. 1. Darla wants to compare the amount of high-fiber nuggets to the total amount of food mix for young adult chimps. She makes this claim:
"High-fiber nuggets are $\frac{3}{2}$ of the total."
Lamar says Darla is wrong. He makes this claim:
"High-fiber nuggets are $\frac{3}{5}$ of the total."
Who is correct? Explain.
2. What fraction of the total amount of food mix for older chimps is high-fiber nuggets?
3. Suppose the ratio of male chimps to female chimps in a zoo is 5 to 4. What fraction of the chimps are male?
4. Suppose $\frac{2}{3}$ of the chimps in a zoo are female. Find the ratio of female chimps to male chimps in that zoo.

$\triangle$ CI Homework starts on page 24.

